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The Impact of Flavour Changing Neutral Gauge Bosons on $\bar{B} \rightarrow X_s \gamma$

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ABSTRACT: The branching ratio of the rare decay $\bar{B} \rightarrow X_s \gamma$ provides potentially strong constraints on models beyond the Standard Model. Considering a general scenario with new heavy neutral gauge bosons, present in particular in Z' and gauge flavour models, we point out two new contributions to the $\bar{B} \rightarrow X_s \gamma$ decay. The first one originates from one-loop diagrams mediated by gauge bosons and heavy exotic quarks with electric charge $-1/3$. The second contribution stems from the QCD mixing of neutral current-current operators generated by heavy neutral gauge bosons and the dipole operators responsible for the $\bar{B} \rightarrow X_s \gamma$ decay. The latter mixing is calculated here for the first time. We discuss general sum rules which have to be satisfied in any model of this type. We emphasise that the neutral gauge bosons in question could also significantly affect other fermion radiative decays as well as non-leptonic two-body B decays, ϵ'/ϵ , anomalous $(g-2)_\mu$ and electric dipole moments.

KEYWORDS: Rare Decays, Beyond Standard Model, Gauge Flavour Symmetries, Z' models

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1 Introduction

Extensions of the Standard Model (SM) with additional gauge symmetries are of particular interest in view of current direct and indirect searches for physics beyond the SM. A feature of such theories is the existence of new gauge bosons. These could provide clear deviations from the SM predictions and have been studied as possible early discoveries at the LHC.

Constructions with a single additional $U(1)$ factor correspond to an extra neutral gauge boson Z' [1–4] and have received a lot of attention, since they are naturally present in a large variety of models, such as Grand Unified Theories (GUTs), higher-dimensional models, little or composite Higgs models, superstring constructions. Hence, the predicted Z' mass spans a wide range of values, from the high GUT scale down to the TeV scale, having different phenomenological consequences. On the other hand, it is difficult to determine model independent bounds on the Z' mass, since its relation to observables depends strongly

on the mixing of the Z' to the SM Z , the strength of the new gauge-coupling constant, the charges of the SM fermions and Higgs doublets, the presence of additional exotic fermions necessary to cancel the anomalies, etc..

Still, there exist bounds on the Z' mass and its mixing with the SM Z from precision electroweak data, direct and indirect searches at the Tevatron, and interference effects at LEP2 in specific Z' models (see [1] and references therein). Assuming an electroweak strength for the new gauge coupling constant and family-universal SM fermion charges, most of the canonical Z' models have predictions in agreement with the experimental constraints, for Z' masses not too much below 1 TeV. However, considering leptophobic Z' models or models with family non-universal SM fermion charges, the Z' mass is allowed to be even smaller.

Beyond the simple $U(1)$ group, extended gauge symmetries have also attracted a lot of attention, especially to explain the observed pattern of fermion masses and mixings (some of the earliest papers on this subject are [5–15]). A common feature of these models is the appearance of dangerous flavour-changing-neutral-current (FCNC) contributions, mediated by the new flavour gauge bosons. Hence, a high new physics (NP) scale, usually larger than 1000 TeV, must be adopted to provide the necessary suppressions. This prevents any NP direct observation at the present and future colliders.

An alternative approach to provide an explanation of the pattern of fermion masses is considering global flavour symmetries; in particular discrete symmetries have been widely studied in the last years (for a review see [16]). It is interesting to notice that the FCNC contributions are usually less constraining in these models [17–27] compared to gauge flavour models, opening the possibility for a lower NP scale and correspondingly for new particles beyond SM that can be detected in present and future colliders.

Recently, a series of papers [28–31] appeared suggesting the possibility of explaining fermion masses and mixings with a NP scale of a few TeV, but with extra gauge symmetries. In these models, the gauge flavour symmetry is the product of non-Abelian $U(3)$ factors. For each gauge-group generator there is a new gauge boson, with no color or electric charge. In addition, new exotic fermions have to be introduced to cancel the anomalies of the new gauge sector.

The analyses in [28–31] and most papers to date studied almost exclusively the impact of flavour changing heavy neutral gauge bosons on $\Delta F = 2$ transitions and rare B and K decays, leaving their impact on the radiative decay $\bar{B} \rightarrow X_s \gamma$ aside. As the latter provides generally a very strong constraint on extensions of the SM, it is important to investigate whether the presence of neutral gauge bosons with flavour violating couplings and new exotic heavy fermions would put the models in question into difficulties and whether generally the role of such new gauge bosons and new fermions in the $\bar{B} \rightarrow X_s \gamma$ decay is significant.

Having this in mind, we point out in this paper two new contributions to the $\bar{B} \rightarrow X_s \gamma$ decay, that to our knowledge have not been considered in the literature so far. The first one originates from one-loop diagrams mediated by *heavy neutral* gauge bosons and *exotic* quarks with electric charge $-1/3$, when *flavour-violating left-handed and right-handed* couplings of these gauge bosons to SM and exotic quarks are present. Then, this new

physics contribution to the Wilson coefficient of the dominant operator is enhanced by m'_F/m_b with m_b being the mass of the b -quark and m'_F the mass of the exotic fermion, in general significantly heavier than SM ones.

Analogous enhancement is known from the study of left-right symmetric models [32–34], but the absence of new heavy fermions and the smallness of $W_L - W_R$ mixing makes this contribution, mediated dominantly by *charged* SM gauge bosons with right-handed couplings, sufficiently small to agree with data.

Within the littlest Higgs model with T-parity, similar contributions to the $\bar{B} \rightarrow X_s \gamma$ decay, mediated by heavy neutral gauge bosons and heavy mirror fermions, have already been considered [35]. However, in this context only left-handed currents are present and in addition the diagrams in question are strongly suppressed by the mixing angles in the T-odd sector and by the GIM mechanism, so that these contributions turn out to be very small.

The second new contribution stems solely from the heavy neutral gauge bosons, independently of the presence of exotic fermions in the theory. These gauge bosons are integrated out at the scale of the order of their masses, in general much higher than $\mu_b \approx m_b$. The resulting *neutral current-current* dimension-six operators affect the $\bar{B} \rightarrow X_s \gamma$ rate through QCD mixing with the magnetic dipole operator responsible for the $\bar{B} \rightarrow X_s \gamma$ decay. This contribution is present when the new neutral gauge bosons have direct couplings to the b and s quarks.

As we shall demonstrate explicitly, these new contributions imply constraints on the parameters of models, containing neutral flavour gauge bosons with couplings described above and with masses within the reach of the LHC. We shall also identify the conditions under which the first of these contributions turns out to be negligible, such as very small mixings among SM and exotic quarks, naturally following from a fermion see-saw mechanism.

We point out that similar contributions could be considered in $f_j \rightarrow f_i \gamma$, like $\mu \rightarrow e \gamma$ and $t \rightarrow c \gamma$, where f is any SM fermion, mediated by exotic fermions with the same electric charge as f . The same comments apply to flavour conserving observables like $(g - 2)_\mu$ and EDMs, in which dipole operators play the dominant role. Moreover, our QCD analysis of the mixing among neutral current-current and QCD-penguin operators is also applicable to other processes, such as non-leptonic two-body B decays, and other observables, like ϵ'/ϵ .

We describe the general framework and fix the notation that will be used throughout the rest of the paper in Sect. 2. In Sect. 3 we present the analytical results for the exotic-fermion contribution to the $\bar{B} \rightarrow X_s \gamma$ decay in the context of only one neutral flavour gauge boson and one exotic quark with electric charge $-1/3$. Subsequently, we generalise our results to an arbitrary number of such gauge bosons and quarks.

In Sect. 4 we discuss the second new contribution to the $\bar{B} \rightarrow X_s \gamma$ decay. We extend the SM QCD renormalisation group (RG) analysis by taking into account all the new neutral current-current operators, generated through the exchange of a single neutral gauge boson A_H . To this end we calculate the relevant anomalous dimension matrices at the leading order (LO).

In Sect. 5 we present our model-independent results in a general form, that can be easily applied to all models with flavour-violating neutral gauge bosons, like Z' -models or models with non-Abelian gauge flavour symmetries. We identify three major classes of such models; for each we cast our results into simple formulae, that depend only on the masses and couplings of the new particles. Finally, we illustrate how these formulae can be translated into constraints on the parameter space of representative toy-models. In Sect. 6 we summarise and conclude.

2 Notation and fermion see-saw mechanism

We focus on models in which the new physics contributions to $\overline{B} \rightarrow X_s \gamma$ discussed in the previous section are present. In what follows, we describe a general framework and we fix the notation.

We consider the general case in which a product of $k \geq 1$ gauge flavour groups, $\mathcal{G}_f = \mathcal{G}_1 \times \dots \times \mathcal{G}_k$, is added to the SM gauge group. Depending on the dimension d_i of each symmetry factor, $d_1 + \dots + d_k$ new flavour gauge bosons enrich the particle spectrum. In general, the SM fermions ψ transform under \mathcal{G}_f and consequently anomalies from gauge or mixed gauge-gravitational triangles can arise. In this case, it is possible to introduce a new set of exotic fermions Ψ , suitably transforming under the SM and gauge flavour groups to obtain an anomaly-free theory.

On the other hand, the presence of these new exotic fermions is a quite general feature of flavour models, when considering renormalisable ultraviolet completions. This holds not only for gauge flavour symmetries, that we are discussing in this paper, but also for global flavour symmetries. Indeed, in these models the new exotic fermions and their mixing with the SM ones account for the mass spectrum of the SM fermions (see [36] and references therein). This typically happens through a fermion see-saw mechanism where the small masses of the SM fermions are explained by the large masses of their corresponding exotic fermions. Without loss of generality, we consider only the case of $SU(2)_L$ -singlet exotic fermions. Hence the see-saw Lagrangian can be schematically written as

$$\mathcal{L}_{\text{see-saw}} = \overline{\Psi}_L M \Psi_R + \overline{\psi}_L Y_1^D \Psi_R \phi + \overline{\Psi}_L M_2^D \psi_R + \text{h.c.}, \quad (2.1)$$

where $\psi_{L,R}$ ($\Psi_{R,L}$) comprise all left- and right-handed SM (exotic) fermion fields; their dimensions also define the dimensions of the matrices M , Y_1^D and M_2^D . In the same expression, ϕ represents the SM Higgs field.

This see-saw Lagrangian can be made invariant under \mathcal{G}_f if both SM and exotic fermions belong to the same representation of \mathcal{G}_f and M , Y_1^D , and M_2^D are proportional to the identity matrix in the flavour space. This case, however, is phenomenologically not interesting, because then the flavour symmetry is exactly preserved and it is not possible to reproduce the observed pattern of SM fermion masses and mixings.

On the other hand, a more realistic situation appears when we impose the invariance under \mathcal{G}_f at high scales, by introducing additional degrees of freedom, and then break the flavour symmetries at lower scales to provide the correct SM fermion masses and mixings. In the see-saw Lagrangian, the invariance under \mathcal{G}_f and its breaking mechanism can be

achieved in various ways: we can promote M , Y_1^D , and M_2^D , or only some of them, either to spurion fields à la MFV [37–40], or to dynamical scalar fields with non-vanishing vacuum expectation values (VEVs) [41], or to fermion condensates [42]. In what follows, we simply treat the terms with M , Y_1^D , and M_2^D , or only some of them, as flavour violating terms originating from some non-specified dynamics at the high scale [43].

In the simplest phenomenologically interesting scenario, we discuss, the fields ψ_L (ψ_R) and Ψ_R (Ψ_L) transform under the same representation of a single flavour group \mathcal{G}_f . The see-saw Lagrangian is the same as in Eq. (2.1), with the only flavour symmetry breaking parameter being M , while Y_1^D and M_2^D are family universal parameters. In general, the square matrix M has non-diagonal hierarchical entries in order to provide the correct description of the SM fermion masses and mixings. Before finding the mass eigenvalues of the SM and exotic fermions, it is useful to diagonalise M . In general, M is diagonalised by a bi-unitary transformation

$$\hat{M} = V_L^\dagger M V_R, \quad (2.2)$$

where $V_{L,R}$ are unitary matrices and \hat{M} is a diagonal matrix. After the absorption of V_L (V_R) in Ψ_L (Ψ_R) and ψ_R (ψ_L) by a field redefinition and the breaking of the electroweak symmetry, the resulting mass matrix is as follows:

$$(\bar{\psi}_L, \bar{\Psi}_L) \begin{pmatrix} 0 & Y_1^D \langle \phi \rangle \times \mathbb{1} \\ M_2^D \times \mathbb{1} & \hat{M} \end{pmatrix} \begin{pmatrix} \psi_R \\ \Psi_R \end{pmatrix}. \quad (2.3)$$

Reabsorbing possible unphysical phases, we can now define the mass eigenstates $f_{L,R}$ and $F_{L,R}$ by performing orthogonal rotations on the flavour eigenstates $\psi_{L,R}$ and $\Psi_{L,R}$. For each generation we may write

$$\begin{pmatrix} f_{L,R} \\ F_{L,R} \end{pmatrix} = \begin{pmatrix} \cos \theta_{L,R} & -\sin \theta_{L,R} \\ \sin \theta_{L,R} & \cos \theta_{L,R} \end{pmatrix} \begin{pmatrix} \psi_{L,R} \\ \Psi_{L,R} \end{pmatrix}. \quad (2.4)$$

Denoting with m_{f_k} the mass of the light fermion f_k and with m'_{F_k} that of the heavy exotic fermion F_k and using for brevity the notation $M_1^D \equiv Y_1^D \langle \phi \rangle$, what follows is a direct inverse proportionality between m_{f_k} and m'_{F_k} , as in the usual see-saw mechanism:

$$m_{f_k} m'_{F_k} = M_1^D M_2^D. \quad (2.5)$$

We can express such masses in terms of the flavour symmetry breaking terms:

$$m_{f_k} = \frac{\sin \theta_{R_k} \sin \theta_{L_k}}{\cos^2 \theta_{R_k} - \sin^2 \theta_{L_k}} \hat{M}_k, \quad m'_{F_k} = \frac{\cos \theta_{R_k} \cos \theta_{L_k}}{\cos^2 \theta_{R_k} - \sin^2 \theta_{L_k}} \hat{M}_k, \quad (2.6)$$

where a straightforward calculation gives

$$\sin \theta_{L_k} = \sqrt{\frac{m_{f_k} M_1^D m'_{F_k} - M_2^D m_{f_k}}{M_2^D (m_{F_k}^{\prime 2} - m_{f_k}^2)}}, \quad \sin \theta_{R_k} = \sqrt{\frac{m_{f_k} M_2^D m'_{F_k} - M_1^D m_{f_k}}{M_1^D (m_{F_k}^{\prime 2} - m_{f_k}^2)}}. \quad (2.7)$$

These results are exact and valid for all the fermion generations. However, taking the limit in which $m'_{F_k} \gg m_{f_k}$ and $M_1^D \approx M_2^D \equiv M^D$, we find simple formulae that transparently expose the behaviour of the previous expressions. In this limit we find

$$m_{f_k} \approx \frac{(M^D)^2}{\hat{M}_k}, \quad m'_{F_k} \approx \hat{M}_k, \quad \sin \theta_{L_k} \approx \sin \theta_{R_k} \approx \sqrt{\frac{m_{f_k}}{m'_{F_k}}}, \quad (2.8)$$

as in the usual see-saw scheme in the limit $\hat{M}_k \gg M_{1,2}^D$. Notice that these simplified relations are valid for all the fermions, apart from the top quark for which the condition $\hat{M}_3 \gg M_{1,2}^D$ is not satisfied and large corrections to Eq. (2.8) are expected.

When the flavour symmetry is broken, the flavour gauge bosons develop masses proportional to the flavour breaking parameters. It is not possible to enter into details without specifying a particular model and in the following we simply assume that the neutral gauge boson masses are controlled by the parameters \hat{M}_k , as for the exotic fermions.

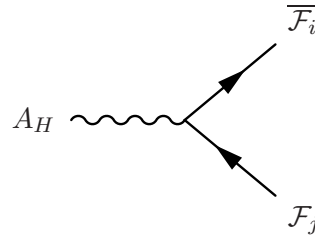
The part of the Lagrangian containing the kinetic terms and the usual couplings among fermion and gauge bosons is given by

$$\mathcal{L}_{kin} = i \left(\bar{\psi} \not{D} \psi + \bar{\Psi} \not{D} \Psi \right). \quad (2.9)$$

Since we are only interested in discussing the couplings between fermions and the new gauge bosons, in writing the relevant covariant derivative we do not consider the coupling terms with the SM gauge bosons but only the following term

$$D^\mu \supset i \frac{g_H}{2} \sum_a A_{H_a}^\mu T_{\mathcal{R}}^a. \quad (2.10)$$

In this expression, g_H is the flavour-gauge coupling, $T_{\mathcal{R}}^a$ are the generators of the flavour group \mathcal{G}_f in the representation \mathcal{R} , and A_{H_a} the corresponding flavour gauge bosons. The resulting Feynman rules for a single gauge boson A_H in the fermion-mass eigenbasis, $\mathcal{F} \equiv \{F, f\}$, are given by the following relations:



$$= i \frac{g_H}{2} \gamma_\mu (P_L C_L + P_R C_R)$$

(2.11)

with

$$A_H \bar{f}_i f_j : \begin{cases} C_L = \cos \theta_{L_i} \left(V_R^\dagger T_{\mathcal{R}}^a V_R \right)_{ij} \cos \theta_{L_j} + \sin \theta_{L_i} \left(V_L^\dagger T_{\mathcal{R}}^a V_L \right)_{ij} \sin \theta_{L_j} \\ C_R = \cos \theta_{R_i} \left(V_L^\dagger T_{\mathcal{R}}^a V_L \right)_{ij} \cos \theta_{R_j} + \sin \theta_{R_i} \left(V_R^\dagger T_{\mathcal{R}}^a V_R \right)_{ij} \sin \theta_{R_j} \end{cases} \quad (2.12)$$

$$A_H \bar{F}_i F_j : \begin{cases} C_L = \sin \theta_{L_i} \left(V_R^\dagger T_{\mathcal{R}}^a V_R \right)_{ij} \sin \theta_{L_j} + \cos \theta_{L_i} \left(V_L^\dagger T_{\mathcal{R}}^a V_L \right)_{ij} \cos \theta_{L_j} \\ C_R = \sin \theta_{R_i} \left(V_L^\dagger T_{\mathcal{R}}^a V_L \right)_{ij} \sin \theta_{R_j} + \cos \theta_{R_i} \left(V_R^\dagger T_{\mathcal{R}}^a V_R \right)_{ij} \cos \theta_{R_j} \end{cases} \quad (2.13)$$

$$A_H \bar{f}_i F_j : \begin{cases} C_L = \cos \theta_{L_i} \left(V_R^\dagger T_{\mathcal{R}}^a V_R \right)_{ij} \sin \theta_{L_j} - \sin \theta_{L_i} \left(V_L^\dagger T_{\mathcal{R}}^a V_L \right)_{ij} \cos \theta_{L_j} \\ C_R = \cos \theta_{R_i} \left(V_L^\dagger T_{\mathcal{R}}^a V_L \right)_{ij} \sin \theta_{R_j} - \sin \theta_{R_i} \left(V_R^\dagger T_{\mathcal{R}}^a V_R \right)_{ij} \cos \theta_{R_j} \end{cases} \quad (2.14)$$

$$A_H \bar{F}_i f_j : \begin{cases} C_L = \sin \theta_{L_i} \left(V_R^\dagger T_{\mathcal{R}}^a V_R \right)_{ij} \cos \theta_{L_j} - \cos \theta_{L_i} \left(V_L^\dagger T_{\mathcal{R}}^a V_L \right)_{ij} \sin \theta_{L_j} \\ C_R = \sin \theta_{R_i} \left(V_L^\dagger T_{\mathcal{R}}^a V_L \right)_{ij} \cos \theta_{R_j} - \cos \theta_{R_i} \left(V_R^\dagger T_{\mathcal{R}}^a V_R \right)_{ij} \sin \theta_{R_j} \end{cases} \quad (2.15)$$

A few comments are in order:

- 1) These results are valid for both Abelian and non-Abelian flavour gauge symmetries. In particular, barring accidental cancellations, the couplings described in Eqs. (2.14) and (2.15) are always non-vanishing. The only exception consists in all those Z' models in which the SM fermions have universal charges. In these cases the flavour gauge symmetry \mathcal{G}_f is a simple $U(1)$ and the charge universality assures that $T_{\mathcal{R}}^a \propto \mathbb{1}$. Hence, the couplings in Eqs. (2.14) and (2.15) identically vanish and the couplings in Eqs. (2.12) and (2.13) turn out to be flavour conserving.
- 2) When the flavour symmetry is a product of different groups, $\mathcal{G}_f = \mathcal{G}_1 \times \dots \times \mathcal{G}_k$, and fermions transform non-trivially under different factors of \mathcal{G}_f , Eqs. (2.12)–(2.15) change. In particular, if ψ_L (Ψ_L) and ψ_R (Ψ_R) transform under two distinct groups, then only one term in the right-hand side of the previous equations is present (for example in Eq. (2.12), C_L could have either the term $\cos \theta_{L_i} \left(V_R^\dagger T_{\mathcal{R}}^a V_R \right)_{ij} \cos \theta_{L_j}$ or $\sin \theta_{L_i} \left(V_L^\dagger T_{\mathcal{R}}^a V_L \right)_{ij} \sin \theta_{L_j}$, but not both terms simultaneously). As a result, even for the simple case $\mathcal{G}_f = U(1)^2$ with universal fermion charges, the couplings described in Eqs. (2.14) and (2.15) are non-vanishing. Notice that in this particular case all the couplings in Eqs. (2.12)–(2.15) turn out to be flavour conserving.
- 3) When the SM fermion masses originate from the see-saw mechanism with heavy exotic fermions, $\sin \theta_{L_k}$ and $\sin \theta_{R_k}$ are small (cf. Eqs. (2.7) and (2.8)). Hence, the couplings described in Eqs. (2.14) and (2.15) are all suppressed. In the particular case of $m'_{F_k} \gg m_{f_k}$, this suppression is roughly given by $\sqrt{m_{f_k}/m'_{F_k}}$.
- 4) When a theory is anomaly free without the necessity of introducing extra exotic fermions, only the expressions in Eq. (2.12) apply. Still the couplings can be flavour violating. This is the case of $U(1)$ models in which either the charge assignment of the SM fermions is such that no anomalies arise or the Green-Schwarz mechanism [44] is implemented into the theory to compensate for the anomalies. In the specific case of Z' models with universal fermion $U(1)$ -charges, the couplings are flavour conserving, as already discussed in 1).

As we will show in Sects. 3 and 4, in virtually any gauge model, in which SM fermion masses are explained through a see-saw mechanism with heavy exotic fermions, both new contributions we introduced in Sect. 1 are present and have potentially interesting effects. Their impact depends on the details of the specific model, according to the above discussion.

On the other hand, for completeness we will also consider models with gauge symmetries beyond those of the SM that provide either an explanation of the fermion mass patterns through a different mechanism than the see-saw or no explanation at all. In these cases, heavy exotic fermions might not be present in the particle spectrum, as in 4), and the corresponding contributions are absent. However, when present, the couplings listed in Eqs. (2.12)–(2.15) have in general a completely different structure. In particular there might be no suppressions of the exotic fermion contributions due to $\sin \theta_{L_k}$ and $\sin \theta_{R_k}$.

The presence of exotic fermions can modify the couplings of the SM W and Z bosons to fermions with respect to their couplings within the SM. As a consequence, non-unitary quark and lepton mixing matrices and flavour violating Z couplings are usual results of such changes, which are specific to the considered gauge flavour model. Also these modifications affect only the Wilson coefficients and do not generate further operators. For this reasons we shall not perform such a study in our model-independent analysis. A study in a model-dependent context considering both SM and heavy gauge bosons contributing to the $\bar{B} \rightarrow X_s \gamma$ branching ratio is in progress [45].

3 New contributions at the high scale

3.1 Effective Hamiltonian for $b \rightarrow s \gamma$

Adopting the overall normalisation of the SM effective Hamiltonian and considering as a first step only the dipole operators and the contribution of new physics to their Wilson coefficients, we write the effective Hamiltonian relevant for $b \rightarrow s \gamma$ at the high matching scale $\mu_H \gg M_W$ as:

$$\mathcal{H}_{\text{eff}}^{(b \rightarrow s \gamma)} = -\frac{4G_F}{\sqrt{2}} V_{ts}^* V_{tb} \left[\Delta C_{7\gamma}(\mu_H) Q_{7\gamma} + \Delta C_{8G}(\mu_H) Q_{8G} + \Delta C'_{7\gamma}(\mu_H) Q'_{7\gamma} + \Delta C'_{8G}(\mu_H) Q'_{8G} \right]. \quad (3.1)$$

At this scale, the W bosons are still dynamical and the usual SM contribution to the Wilson coefficients is absent. We will include it after integrating out the W boson at the lower scale $\mu_W \approx \mathcal{O}(M_W)$. In our conventions the dipole operators are given by:

$$\begin{aligned} Q_{7\gamma} &= \frac{e}{16\pi^2} m_b \bar{s}_\alpha \sigma^{\mu\nu} P_R b_\alpha F_{\mu\nu}, \\ Q_{8G} &= \frac{g_s}{16\pi^2} m_b \bar{s}_\alpha \sigma^{\mu\nu} P_R T_{\alpha\beta}^a b_\beta G_{\mu\nu}^a \end{aligned} \quad (3.2)$$

and the primed operators are obtained from them after replacing the right-handed projector, P_R , with the left-handed one, P_L . In the SM the contributions of the primed operators are suppressed by m_s/m_b relative to those coming from $Q_{7\gamma}$ and Q_{8G} . We decompose the Wilson coefficients at the scale μ_W as the sum of the SM contribution and the new one from the exchange of neutral flavour gauge bosons, after evolving the new physics contribution using the RG running of the operators $Q_{7\gamma}^{(\prime)}$ and $Q_{8G}^{(\prime)}$, namely

$$C_i(\mu_W) = C_i^{SM}(\mu_W) + \Delta C_i(\mu_W) \quad (3.3)$$

and similarly for the primed coefficients. The SM Wilson coefficients at LO are [46]

$$C_{7\gamma}^{SM}(\mu_W) = \frac{3x_t^3 - 2x_t^2}{4(x_t - 1)^4} \ln x_t - \frac{8x_t^3 + 5x_t^2 - 7x_t}{24(x_t - 1)^3}, \quad (3.4)$$

$$C_{8G}^{SM}(\mu_W) = \frac{-3x_t^2}{4(x_t - 1)^4} \ln x_t - \frac{x_t^3 - 5x_t^2 - 2x_t}{8(x_t - 1)^3}, \quad (3.5)$$

where $x_t \equiv m_t^2/M_W^2$. The corresponding primed coefficients are also given by the previous expressions, but with an extra suppression factor of m_s/m_b .

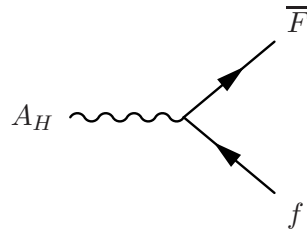
The new physics contributions at μ_W , $\Delta C_i(\mu_W)$, follow from the RG evolution of the Wilson coefficients at μ_H , $\Delta C_i(\mu_H)$, down to μ_W ; Their relations are discussed in Sect. 4. We can further decompose the new physics contributions at the high scale as the sum of two pieces, one coming from the exchanges of heavy exotic quarks and the second from the exchanges of light SM down-quarks: respectively,

$$\begin{aligned} \Delta C_{7\gamma}(\mu_H) &= \Delta C_{7\gamma}^{\text{heavy}}(\mu_H) + \Delta C_{7\gamma}^{\text{light}}(\mu_H), \\ \Delta C_{8G}(\mu_H) &= \Delta C_{8G}^{\text{heavy}}(\mu_H) + \Delta C_{8G}^{\text{light}}(\mu_H). \end{aligned} \quad (3.6)$$

In the following we analyse these two contributions separately.

3.2 Contributions from heavy exotic quark exchanges

We begin with the new contributions to $b \rightarrow s\gamma$ arising from the exchange of heavy neutral flavour gauge bosons and heavy exotic quarks. In what follows, we restrict the discussion to the contribution from a single virtual gauge boson A_H and a single virtual heavy quark of electric charge $-1/3$ and mass m'_F . The results will then be generalised to an arbitrary number of such particles. From the general Feynman rules listed in Eqs. (2.14) and (2.15), we read off the flavour changing couplings of A_H to the bottom and strange quarks and the exotic quark F relevant for $b \rightarrow s\gamma$:



$$= i \frac{g_H}{2} \gamma_\mu \left(P_L C_L^{fF} + P_R C_R^{fF} \right) \quad (3.7)$$

The only diagram¹ with a virtual F and A_H exchange contributing to the $b \rightarrow s\gamma$ transition in the unitary gauge is shown in Fig. 1. In the same figure, we also show the analogous diagram contributing to $b \rightarrow sg$. The latter will also contribute to the $\bar{B} \rightarrow X_s\gamma$ rate via QCD mixing of Q_{8G} in $Q_{7\gamma}$. In both diagrams, there is no triple gauge boson vertex as the heavy gauge boson A_H considered does not carry electric or colour charge.

¹The corrections on the external legs, necessary for a canonical kinetic term, are understood to be contained in finite off-diagonal field renormalisations.

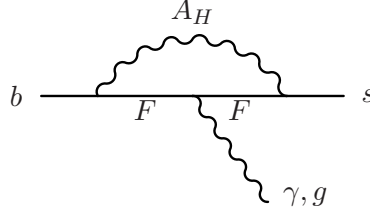


Figure 1. Magnetic penguin diagram contributing to $b \rightarrow s\gamma$ and $b \rightarrow sg$ with internal A_H and F exchanges.

We further decompose the Wilson coefficients $\Delta C_i^{\text{heavy}}$ at the high scale as the sum of the SM-like LL contribution and a new LR one, where L (R) stands for the P_L (P_R) projector in the vertex of Eq. (3.7) involving the $s(b)$ -quark:

$$\begin{aligned}\Delta C_{7\gamma}^{\text{heavy}}(\mu_H) &= \Delta^{LL} C_{7\gamma}^{\text{heavy}}(\mu_H) + \Delta^{LR} C_{7\gamma}^{\text{heavy}}(\mu_H), \\ \Delta C_{8G}^{\text{heavy}}(\mu_H) &= \Delta^{LL} C_{8G}^{\text{heavy}}(\mu_H) + \Delta^{LR} C_{8G}^{\text{heavy}}(\mu_H).\end{aligned}\tag{3.8}$$

The corresponding primed coefficients to Eq. (3.8) are obtained by interchanging $L \leftrightarrow R$ and including the suppression factor m_s/m_b in the primed Wilson coefficients.

Following [35], the SM-like contributions LL and RR are obtained by noting that their topological structure is the same as the SM *gluon* magnetic penguin. Consequently, the known SM results for the Wilson coefficient of the gluon dipole operator, C_{8G}^{SM} , can be used to obtain both LL and RR contributions after a suitable redefinition of couplings and masses. Notice that in the neutral exchange there is generally no analogue of a GIM mechanism, therefore mass independent terms in the SM Wilson coefficients must be kept. We extract them from [47, 48]. Our results are as follows:

$$\begin{aligned}\Delta^{LL} C_{7\gamma}^{\text{heavy}}(\mu_H) &= -\frac{1}{6} \frac{g_H^2}{g_2^2} \frac{M_W^2}{M_{A_H}^2} \frac{C_L^{sF*} C_L^{bF}}{V_{ts}^* V_{tb}} \left(C_{8G}^{SM}(x) + \frac{1}{3} \right), \\ \Delta^{LL} C_{8G}^{\text{heavy}}(\mu_H) &= -3 \Delta^{LL} C_{7\gamma}^{\text{heavy}}(\mu_H),\end{aligned}\tag{3.9}$$

with $x = m_F^2/M_{A_H}^2$. The RR contributions are obtained by the interchange $L \leftrightarrow R$. $C_{8G}^{SM}(x)$ is the SM function on the right-hand side of Eq. (3.5).

We now consider the LR and RL contributions, which have no SM equivalent. To obtain the Wilson coefficients, we adapt to our case known calculations of $b \rightarrow sg$ in the context of $b \rightarrow s\gamma$ in the left-right symmetric models [34, 47], where, however, the process is mediated by charged gauge bosons. In this way, we obtain the following LR Wilson coefficients:

$$\begin{aligned}\Delta^{LR} C_{7\gamma}^{\text{heavy}}(\mu_H) &= -\frac{1}{6} \frac{g_H^2}{g_2^2} \frac{M_W^2}{M_{A_H}^2} \frac{m_F'}{m_b} \frac{C_L^{sF*} C_R^{bF}}{V_{ts}^* V_{tb}} C_{8G}^{LR}(x), \\ \Delta^{LR} C_{8G}^{\text{heavy}}(\mu_H) &= -3 \Delta^{LR} C_{7\gamma}^{\text{heavy}}(\mu_H),\end{aligned}\tag{3.10}$$

with

$$C_{8G}^{LR}(x) = \frac{-3x}{2(1-x)^3} \ln x + \frac{3x(x-3)}{4(x-1)^2} - 1.\tag{3.11}$$

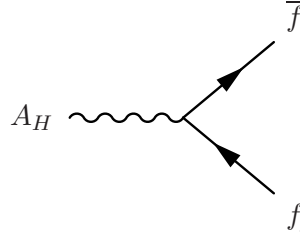
The analogous RL contributions are obtained by interchanging $L \leftrightarrow R$. In this case $C_{8G}^{RL}(x) = C_{8G}^{LR}(x)$.

The following properties should be noted:

- 1) the LR (RL) contribution in Eq. (3.10) dominates over the LL (RR) one of Eq. (3.9) in a large part of the parameter space, due to the factor m'_F/m_b ;
- 2) the factor m'_F/m_b is replaced by m_t/m_b in the usual left-right symmetric models, therefore in these models the enhancement is less pronounced;
- 3) $C_{8G}^{LR}(x)$ is a non-vanishing monotonic function of x and takes values in the range $[-1, -1/4]$ for x from 0 to ∞ ;
- 4) in the decoupling limit these contributions turn out to vanish. Indeed when large masses for the exotic fermions m'_{F_k} are considered, the masses of the neutral gauge bosons M_{A_H} are also large, as we have already commented in Sect. 2. Therefore the LL (RR) contributions approach zero with $1/M_{A_H}^2$ and the LR (RL) ones are strongly suppressed by $m'_F/M_{A_H}^2$.

3.3 Contributions from light SM quark exchanges

We now discuss $\Delta C_{7\gamma}^{\text{light}}$ and $\Delta C_{8G}^{\text{light}}$ from Eq. (3.6), namely the contributions arising from the exchange of light down-type quarks in loop diagrams analogous to the one in Fig 1, where the flavour changing couplings of A_H to the SM quarks read



$$= i \frac{g_H}{2} \gamma_\mu \left(P_L C_L^{f_j f_i} + P_R C_R^{f_j f_i} \right)$$

(3.12)

arising from the general Feynman rules listed in Eq. (2.12). In the SM, the light-quark contributions cancel each other at scales well above their masses due to the GIM mechanism. This is in contrast to the present case, where such a mechanism is absent.

Following the procedure of the previous section, we decompose the Wilson coefficients at the high scale as the sum of a LL part and a LR part:

$$\begin{aligned} \Delta C_{7\gamma}^{\text{light}}(\mu_H) &= \Delta^{LL} C_{7\gamma}^{\text{light}}(\mu_H) + \Delta^{LR} C_{7\gamma}^{\text{light}}(\mu_H), \\ \Delta C_{8G}^{\text{light}}(\mu_H) &= \Delta^{LL} C_{8G}^{\text{light}}(\mu_H) + \Delta^{LR} C_{8G}^{\text{light}}(\mu_H). \end{aligned} \tag{3.13}$$

In the framework of effective field theories the light degrees of freedom are treated as massless at the high matching-scale μ_H . Therefore, we only need to account for the first term in the expansion of the light masses and only mass-independent terms in Eq. (3.9)

and (3.10) contribute. For the LL SM-like contributions we find

$$\Delta^{LL} C_{7\gamma}^{\text{light}}(\mu_H) = -\frac{1}{6} \frac{g_H^2}{g_2^2} \frac{M_W^2}{M_{A_H}^2} \sum_f \frac{C_L^{sf*} C_L^{bf}}{V_{ts}^* V_{tb}} \left(\frac{1}{3} \right), \quad (3.14)$$

$$\Delta^{LL} C_{8G}^{\text{light}}(\mu_H) = -3\Delta^{LL} C_{7\gamma}^{\text{light}}(\mu_H),$$

and for the LR contribution we have from Eqs. (3.10) and (3.11)

$$\Delta^{LR} C_{7\gamma}^{\text{light}}(\mu_H) = -\frac{1}{6} \frac{g_H^2}{g_2^2} \frac{M_W^2}{M_{A_H}^2} \sum_f \frac{m_f}{m_b} \frac{C_L^{sf*} C_R^{bf}}{V_{ts}^* V_{tb}} (-1), \quad (3.15)$$

$$\Delta^{LR} C_{8G}^{\text{light}}(\mu_H) = -3\Delta^{LR} C_{7\gamma}^{\text{light}}(\mu_H),$$

where f stands for the SM down-type quarks. Analogously, we obtain also the results for the primed Wilson coefficients. The LL and RR SM-like contributions have already been considered in [49], while the LR and RL are new. Notice that the latter come with a factor of m_f/m_b and hence, the effects from d - and s -quarks are suppressed with respect to the b -quark contribution.

3.4 Generalisations and the GIM-like mechanism

It is easy to extend the previous results to the case of an arbitrary number of neutral flavour gauge bosons, A_{H_i} , and heavy fermions, F_k , by performing the following substitution in Eqs. (3.9) and (3.10):

$$\frac{g_H^2}{M_{A_H}^2} C_{L,R}^{sF*} C_{L,R}^{bF} \left(C_{8G}^{SM}(x) + \frac{1}{3} \right) \longrightarrow \sum_{i,k} \frac{g_{H_i}^2}{M_{A_{H_i}}^2} C_{L,R}^{ski*} C_{L,R}^{bki} \left(C_{8G}^{SM}(x_{ki}) + \frac{1}{3} \right), \quad (3.16)$$

$$g_H^2 \frac{m'_F}{M_{A_H}^2} C_{L,R}^{sF*} C_{R,L}^{bF} C_{8G}^{LR}(x) \longrightarrow \sum_{i,k} g_{H_i}^2 \frac{m'_{F_k}}{M_{A_{H_i}}^2} C_{L,R}^{ski*} C_{R,L}^{bki} C_{8G}^{LR}(x_{ki}),$$

where $x_{ki} \equiv m'^2_{F_k}/M_{A_{H_i}}^2$ and $C_{L,R}^{(s,b)ki}$ are the couplings among the light quarks (s, b) with A_{H_i} and F_k . In this substitution we take A_{H_i} to be the mass eigenstates of the neutral gauge bosons. Similarly, we may also generalise Eq. (3.14) and (3.15).

Notice that in the case that the masses of the heavy particles span a wide range of values, the generalisation above is only a first approximation: a more precise result follows from a detailed analysis using the results of Sect. 4 considering several threshold scales. This further study goes beyond the scope of the present paper and in the following we only consider a single high matching scale.

The factor $m'_{F_k}/M_{A_{H_i}}^2$ can represent a dangerous enhancement of the LR (RL) contribution in Eq. (3.16), which is unlikely to vanish, barring accidental cancellations. We identify a series of conditions under which this contribution exactly vanishes:

- 1) $g_{H_i} = g_H$, i.e. there is only one gauge symmetry or all the new gauge coupling constants have the same strength;

- 2) $m'_{F_k} = m'_F$, $M_{A_{H_i}} = M_{A_H}$, i.e. all the exotic fermions and the neutral gauge bosons are degenerate in mass, as in the case of unbroken flavour symmetries;
- 3) $\sum_i C_{L,R}^{ski*} C_{L,R}^{bki} = \sum_i C_{L,R}^{ski*} C_{R,L}^{bki} = 0$, i.e. the couplings are unitary matrices satisfying $C_L = C_R$.

Similar conditions apply also to the contributions of SM down-type quarks.

These conditions assure the exact cancellation of the NP contributions. However, similarly to the GIM mechanism, if they are only partially satisfied, the terms in Eq. (3.16) do not cancel each other completely and the NP contributions could still be dangerous.

Furthermore, we remark that condition 2) would correspond to degenerate SM fermions and thus these conditions cannot be satisfied in models which successfully explain the SM fermion mass pattern through the see-saw mechanism illustrated in Sect. 2. In these models, however, the mixings between SM and exotic fermions are small, cf. Eqs. (2.14) and (2.15). This provides a sufficiently strong suppression to safely neglect these contributions. We further discuss this point in Sect. 5.2.

4 QCD corrections

4.1 General structure

In order to complete the analysis of the $b \rightarrow s\gamma$ decay we have to include QCD corrections, which play a very important role within the SM, enhancing the rate by a factor of 2–3 [50]. It originates dominantly from the mixing of *charged* current-current operators into the dipole operators and to a smaller extent from the mixing with QCD-penguin operators.

When the flavour gauge bosons A_{H_i} and the exotic fermions F_k are integrated out at the high scale μ_H , in addition to the dipole operators discussed at length in the previous section, also *neutral* current-current operators corresponding to a tree-level A_{H_i} exchange are generated together with ordinary QCD- and QED-penguin operators.

The contributions of QCD- and QED-penguin operators, arising from diagrams with A_{H_i} and F_k exchanges, are much less important than within the SM. Indeed, in the SM context, QCD-penguins do not have to compete with tree-level FCNC diagrams, as the latter are absent due to the GIM mechanism. However, in the present case there are neutral current-current operators originating from the tree-level A_{H_i} exchanges that are not suppressed by $\alpha_s(\mu_H)$ and loop effects, contrary to QCD-penguins. Thus, for all practical purposes the contributions of QCD-penguins from A_{H_i} and F_k exchanges at the high scale μ_H can be neglected. Even less important are effects from QED-penguins. Note that this is in contrast to dipole operators which cannot be generated at tree-level, but nevertheless mediate the $b \rightarrow s\gamma$ decay.

In what follows we will extend the SM RG analysis by considering the QCD effects of neutral current-current operators at scales $\mu_b \leq \mu \leq \mu_H$ generated by diagrams with the exchange of a single neutral gauge boson A_H . The extension to the case of an arbitrary number of such gauge bosons is straightforward. This analysis does take into account the mixing under QCD renormalisation of these neutral current-current operators 1) into

dipole operators, 2) among themselves, 3) into QCD-penguin operators. The latter mixing generates contributions to QCD-penguin operators at scales lower than μ_H , even if the initial conditions at μ_H of the Wilson coefficients of these operators are neglected. Even if this effect is numerically negligible, we include it for completeness as the mixing of QCD-penguin operators and dipole operators is taken into account in the SM part.

Before going into details, let us note that the neutral current-current operators do not mix with the charged current-current operators as their flavour structures differ from each other. On the other hand, similarly to charged current-current operators, neutral current-current operators have an impact on dipole operators and QCD-penguins through RG effects without being themselves affected by the presence of these two types of operators.

We would like to emphasise that the QCD analysis presented below is also relevant for other processes, such as non-leptonic two-body B decays, and other observables, like ϵ'/ϵ .

Denoting symbolically charged current-current, QCD-penguin, dipole, and neutral current-current operators and the corresponding primed operators respectively by

$$Q^{cc}, \quad Q_P, \quad Q_D, \quad Q^{nn}, \quad Q'_P, \quad Q'_D, \quad Q^{nn'}, \quad (4.1)$$

the structure of the one-loop anomalous dimension matrix looks as follows:

$$\begin{array}{c}
 \begin{array}{c} Q^{cc} \\ Q_P \\ Q_D \\ Q^{nn} \end{array}
 \begin{array}{|c|c|c|c|}
 \hline
 X_1 & X_2 & X_3 & 0 \\
 \hline
 0 & X_4 & X_5 & 0 \\
 \hline
 0 & 0 & X_6 & 0 \\
 \hline
 0 & Y_1 & Y_2 & Y_3 \\
 \hline
 \end{array}
 \begin{array}{|c|c|c|}
 \hline
 X_4 & X_5 & 0 \\
 \hline
 0 & X_6 & 0 \\
 \hline
 Y_1 & Y_2 & Y_3 \\
 \hline
 \end{array}
 \begin{array}{c} Q'_P \\ Q'_D \\ Q^{nn'} \end{array}
 \end{array} \quad (4.2)$$

The non-vanishing entries denoted by X_i are known from the SM analysis [51, 52]. The new results in the present paper are the entries denoted by Y_i , the initial conditions for Q_D , given already in Sect. 3, and the corresponding conditions for Q^{nn} , given at the end of this section.

The RG analysis of the NP contributions can be performed independent from the SM one: that is

$$\begin{aligned}
 C_i^{SM}(\mu_b) &= U_{ij}(\mu_b, \mu_W) C_j^{SM}(\mu_W), \\
 C_i^{NP}(\mu_b) &= W_{ij}(\mu_b, \mu_H) C_j^{NP}(\mu_H), \\
 C_i^{\text{total}}(\mu_b) &= C_i^{SM}(\mu_b) + C_i^{NP}(\mu_b),
 \end{aligned} \quad (4.3)$$

where U_{ij} and W_{ij} are the elements of the RG evolution matrix. The Wilson coefficients for the primed operators at the high scale are in general different from the expression above, but the evolution matrices U and W do not change. The general structure of these matrices in terms of the anomalous dimension matrices and the QCD- β -functions is well known [53] and will not be repeated here. Therefore, we present in the next sections our results for the matrices Y_i and the initial conditions for Q^{nn} and $Q^{nn'}$ operators.

4.2 Operator basis for neutral current-current operators

There are 48 neutral current-current operators generated by the exchange of A_H , all containing the neutral currents $(\bar{s} \gamma_\mu P_{L,R} b)$ and the flavour conserving currents $(\bar{f} \gamma_\mu P_{L,R} f)$ with $f = u, c, t, d, s, b$. At this scale also the exotic fermions have been already integrated out at μ_H .

The general notation for the 48 Q^{nn} and $Q^{nn'}$ operators in question will be

$$Q_{1,2}^f(A, B), \quad \text{for } A, B = \{L, R\}. \quad (4.4)$$

For instance

$$\begin{aligned} Q_1^u(L, R) &= (\bar{s}_\alpha \gamma_\mu P_L b_\beta) (\bar{u}_\beta \gamma^\mu P_R u_\alpha), \\ Q_2^u(L, R) &= (\bar{s}_\alpha \gamma_\mu P_L b_\alpha) (\bar{u}_\beta \gamma^\mu P_R u_\beta), \end{aligned} \quad (4.5)$$

where $\alpha, \beta = 1, 2, 3$ are colour indices.

Moreover there are 8 additional neutral current-current operators that have to be included, as they have a different structure, when colour and flavour structures are considered simultaneously:

$$\begin{aligned} \hat{Q}_1^d(A, B) &= (\bar{s}_\alpha \gamma_\mu P_A d_\beta) (\bar{d}_\beta \gamma^\mu P_B b_\alpha), \\ \hat{Q}_2^d(A, B) &= (\bar{s}_\alpha \gamma_\mu P_A d_\alpha) (\bar{d}_\beta \gamma^\mu P_B b_\beta). \end{aligned} \quad (4.6)$$

In this classification, we refer to Q^{nn} [$Q^{nn'}$] as those operators $Q_{1,2}(A, B)$ with $A = L$ [$A = R$] and $B = L, R$ [$B = R, L$].

We point out that the operator basis in Eqs. (4.4) and (4.6) can be reduced using Fierz transformations. Moreover some relations can be found between the usual charged current-current and QCD-penguin operators

$$\begin{aligned} Q_1 &= (\bar{s}_\alpha \gamma_\mu P_L c_\beta) (\bar{c}_\beta \gamma^\mu P_L b_\alpha), \\ Q_2 &= (\bar{s}_\alpha \gamma_\mu P_L c_\alpha) (\bar{c}_\beta \gamma^\mu P_L b_\beta), \\ Q_3 &= (\bar{s}_\alpha \gamma_\mu P_L b_\alpha) \sum_{q=u,d,s,c,b} (\bar{q}_\beta \gamma^\mu P_L q_\beta), \\ Q_4 &= (\bar{s}_\alpha \gamma_\mu P_L b_\beta) \sum_{q=u,d,s,c,b} (\bar{q}_\beta \gamma^\mu P_L q_\alpha), \\ Q_5 &= (\bar{s}_\alpha \gamma_\mu P_L b_\alpha) \sum_{q=u,d,s,c,b} (\bar{q}_\beta \gamma^\mu P_R q_\beta), \\ Q_6 &= (\bar{s}_\alpha \gamma_\mu P_L b_\beta) \sum_{q=u,d,s,c,b} (\bar{q}_\beta \gamma^\mu P_R q_\alpha). \end{aligned} \quad (4.7)$$

Yet, similarly to [54, 55] we have decided to work with all operators and not use such relations when performing the RG analysis in order to keep the anomalous dimensions in a transparent form. As discussed in [54, 55] one can work with linearly dependent operators in the process of RG analysis without any problems and use such relations only at the end of the evolution if necessary. However, in the case of $C_{7\gamma}(\mu_b)$ at LO this is not required.

In this context we should warn the reader that at NLO level a use of Fierz relations in the RG analysis could lead to wrong results within the NDR scheme unless Fierz evanescent operators are introduced [56, 57].

4.3 Anomalous dimension matrices

We define the LO anomalous dimension matrix through

$$\hat{\gamma}(\alpha_s) = \frac{\alpha_s}{4\pi} \hat{\gamma}^{(0)}, \quad (4.8)$$

and give below the results for $\hat{\gamma}^{(0)}$ (dropping the index (0) to simplify the notation).

The inspection of the one-loop diagrams contributing to the anomalous dimension matrices shows that only 16 operators have to be considered in order to find the full matrix. These are

$$\begin{aligned} & Q_{1,2}^u(L, L), \quad Q_{1,2}^d(L, L), \quad Q_{1,2}^s(L, L), \quad \hat{Q}_{1,2}^d(L, L), \\ & Q_{1,2}^u(L, R), \quad Q_{1,2}^d(L, R), \quad Q_{1,2}^s(L, R), \quad \hat{Q}_{1,2}^d(L, R). \end{aligned} \quad (4.9)$$

The anomalous dimensions for u replaced by c and t are equal to the one of $Q_{1,2}^u$. The ones of $Q_{1,2}^b$ are equal to the ones of $Q_{1,2}^s$. The anomalous dimensions of the remaining 28 operators, namely the primed operators obtained by $L \leftrightarrow R$, are the same as those of the corresponding unprimed operators (see Eq. (4.2)).

The mixing between the Q^{nn} operators and Q_D , that is the matrix Y_2 , can be extracted from [51, 58] by inspecting the mixing between QCD-penguin and dipole operators. For the *transposed* matrices we get

$$\hat{\gamma}_D^T(L, L) = \begin{array}{c|ccccccccc} & Q_1^u & Q_2^u & Q_1^d & Q_2^d & Q_1^s & Q_2^s & \hat{Q}_1^d & \hat{Q}_2^d \\ \hline Q_{7\gamma} & \frac{416}{81} & 0 & -\frac{232}{81} & 0 & -\frac{232}{81} & -\frac{232}{81} & 0 & -\frac{232}{81} \\ \hline Q_{8G} & \frac{70}{27} & 3 & \frac{70}{27} & 3 & \frac{151}{27} & \frac{151}{27} & 3 & \frac{70}{27} \end{array} \quad (4.10)$$

$$\hat{\gamma}_D^T(L, R) = \begin{array}{c|ccccccccc} & Q_1^u & Q_2^u & Q_1^d & Q_2^d & Q_1^s & Q_2^s & \hat{Q}_1^d & \hat{Q}_2^d \\ \hline Q_{7\gamma} & -\frac{448}{81} & 0 & \frac{200}{81} & 0 & \frac{200}{81} & \frac{16}{9} & -\frac{80}{3} & -\frac{32}{9} \\ \hline Q_{8G} & -\frac{119}{27} & -3 & -\frac{119}{27} & -3 & -\frac{173}{27} & -\frac{16}{3} & -4 & \frac{8}{3} \end{array} \quad (4.11)$$

The mixing between neutral current-current operators, that is the matrix Y_3 , is universally given by two 2×2 matrices:

$$\hat{\gamma}^{nn}(L, L) = \begin{array}{c|cc} & Q_1 & Q_2 \\ \hline Q_1 & -2 & 6 \\ \hline Q_2 & 6 & -2 \end{array} \quad \hat{\gamma}^{nn}(L, R) = \begin{array}{c|cc} & Q_1 & Q_2 \\ \hline Q_1 & -16 & 0 \\ \hline Q_2 & -6 & 2 \end{array} \quad (4.12)$$

Only operators with the same flavour-conserving structure $(\bar{f} \gamma_\mu P_{L,R} f)$ mix in this way.

Finally, in the case of Y_1 there is a universal mixing of Q^{nn} operators into QCD-penguin operators:

$$\hat{\gamma}_P = \begin{array}{c|cccc} & Q_3 & Q_4 & Q_5 & Q_6 \\ \hline Q^{nn} & -\frac{2}{9} & \frac{2}{3} & -\frac{2}{9} & \frac{2}{3} \end{array} \quad (4.13)$$

for

$$Q^{nn} = \left\{ Q_1^{u,c,t,d,s,b}(L, L), Q_2^{s,b}(L, L), \hat{Q}_2^d(L, L), Q_1^{u,c,t,d,s,b}(L, R) \right\} \quad (4.14)$$

with no mixing for the remaining operators in Eq. (4.9).

4.4 Initial conditions

The initial conditions at μ_H for the dipole operators have already been presented in Sect. 3: see Eqs. (3.4), (3.5), (3.9), (3.10), (3.14) and (3.15). Here we give the corresponding initial conditions for the neutral current-current operators.

The initial conditions are determined by integrating out all the heavy degrees of freedom at μ_H and matching the result to the effective theory. At LO this matching is trivial. In the following we give the result for the simplified case in which only one neutral flavour gauge boson A_H contributes to the initial conditions. The generalisation to several gauge bosons is obvious.

Denoting the vertex $\bar{f}_i A_H f_j$ as in Eqs. (2.11) and (2.12), the general expression for the Wilson coefficient of the neutral current-current operator

$$Q_2^f(A, B) \equiv (\bar{s}_\alpha \gamma_\mu P_A b_\alpha)(\bar{f}_\beta \gamma^\mu P_B f_\beta), \quad (4.15)$$

in the normalisation of Eq. (3.1), is given by

$$\Delta^{AB} C_2^f(\mu_H) = -\frac{1}{2} \frac{g_H^2}{g_2^2} \frac{M_W^2}{M_{A_H}^2} \frac{C_A^{sb*} C_B^{ff}}{V_{ts}^* V_{tb}}, \quad (4.16)$$

while all the coefficients $\Delta^{AB} C_1$ are zero. Similarly, the Wilson coefficient for the $\hat{Q}_2^d(A, B)$ operator is

$$\Delta^{AB} \hat{C}_2^d(\mu_H) = -\frac{1}{2} \frac{g_H^2}{g_2^2} \frac{M_W^2}{M_{A_H}^2} \frac{C_A^{sd*} C_B^{bd}}{V_{ts}^* V_{tb}}, \quad (4.17)$$

while $\Delta^{AB} \hat{C}_1^d(\mu_H)$ is again zero.

5 Phenomenological analysis

5.1 The $\bar{B} \rightarrow X_s \gamma$ branching ratio

The SM prediction for the $\bar{B} \rightarrow X_s \gamma$ branching ratio at NNLO [50, 59, 60] reads,

$$Br(\bar{B} \rightarrow X_s \gamma) = (3.15 \pm 0.23) \times 10^{-4}, \quad (5.1)$$

and has been calculated for a photon-energy cut-off $E_\gamma > 1.6$ GeV in the \bar{B} -meson rest frame. This is to be compared with the current experimental value [61],

$$Br(\bar{B} \rightarrow X_s \gamma) = (3.55 \pm 0.24 \pm 0.09) \times 10^{-4}, \quad (5.2)$$

for the same energy cut-off E_γ . In the presence of NP the expression for the branching ratio is given as follows:

$$Br(\bar{B} \rightarrow X_s \gamma) = R (|C_{7\gamma}(\mu_b)|^2 + |C'_{7\gamma}(\mu_b)|^2 + N(E_\gamma)) , \quad (5.3)$$

where $R = 2.47 \times 10^{-3}$ and $N(E_\gamma) = (3.6 \pm 0.6) \times 10^{-3}$. The parameter R is simply an overall factor, whose determination is discussed in [59, 60], while $N(E_\gamma)$ is a non-perturbative contribution. Calculating the NP contributions in the LO approximation, but including the SM contribution at the NNLO level we find for $\mu_b = 2.5$ GeV

$$C_{7\gamma}(\mu_b) = C_{7\gamma}^{SM}(\mu_b) + \Delta C_{7\gamma}(\mu_b) \quad (5.4)$$

where the central value of $C_{7\gamma}^{SM}(\mu_b)$ at the NNLO, corresponding to the value in Eq. (5.1), is given by [50, 59, 60]

$$C_{7\gamma}^{SM}(\mu_b) = -0.3523 \quad (5.5)$$

and the NP one, calculated by us at the LO, by

$$\begin{aligned} \Delta C_{7\gamma}(\mu_b) = & \kappa_7 \Delta C_{7\gamma}(\mu_H) + \kappa_8 \Delta C_{8G}(\mu_H) + \\ & + \sum_{\substack{A=L,R \\ f=u,c,t,d,s,b}} \kappa_{LA}^f \Delta^{LA} C_2^f(\mu_H) + \sum_{A=L,R} \hat{\kappa}_{LA}^d \Delta^{LA} \hat{C}_2^d(\mu_H). \end{aligned} \quad (5.6)$$

Here κ 's are the NP magic numbers listed in Tab. 1, calculated taking $\alpha_s(M_Z = 91.1876 \text{ GeV}) = 0.118$. The primed coefficient $C'_{7\gamma}(\mu_b)$ can be obtained from (5.4)–(5.6), by interchanging $L \leftrightarrow R$ and taking the initial conditions of the primed Wilson coefficients. In particular, the NP magic numbers listed in Tab. 1 are also valid for the primed case, as QCD is blind to the fermion chirality².

The SM contribution containing NLO and NNLO QCD corrections exhibits a negligible μ_b -dependence. This is not the case for the NP contribution at LO. However, we have checked numerically that when the NP contribution enhances the SM value by 20%, the μ_b -dependence in the total branching ratio amounts to a 3% uncertainty for $\mu_b \in [2.5, 5]$ GeV. For smaller deviations from the SM prediction the uncertainty further decreases. This renders this uncertainty sufficiently small for our purposes.

Before concluding a few observations are in order:

²In the computation of $C'_{7\gamma}(\mu_b)$ we are neglecting the SM contributions $C_{7\gamma}^{SM'}(\mu_b)$ and $C_{8G}^{SM'}(\mu_b)$, and the NP contributions $\Delta C'_{7\gamma}(\mu_b)$ and $\Delta C'_{8G}(\mu_b)$, since all of them are suppressed by m_s/m_b with respect to the unprimed ones. Our numerical analyses confirm that this approximation is consistent with errors below the percent level.

μ_H	200 GeV	1 TeV	5 TeV	10 TeV	M_Z
κ_7	0.524	0.457	0.408	0.390	0.566
κ_8	0.118	0.125	0.129	0.130	0.111
$\kappa_{LL}^{u,c}$	0.039	0.057	0.076	0.084	0.030
κ_{LL}^t	-0.002	-0.003	-0.002	-0.001	—
κ_{LL}^d	-0.040	-0.057	-0.072	-0.079	-0.032
$\kappa_{LL}^{s,b}$	0.087	0.090	0.090	0.090	0.084
$\hat{\kappa}_{LL}^d$	0.128	0.147	0.163	0.168	0.116
$\kappa_{LR}^{u,c}$	0.085	0.128	0.173	0.193	0.065
κ_{LR}^t	0.004	0.012	0.023	0.028	—
κ_{LR}^d	-0.015	-0.025	-0.036	-0.041	-0.011
$\kappa_{LR}^{s,b}$	-0.078	-0.092	-0.106	-0.111	-0.070
$\hat{\kappa}_{LR}^d$	0.473	0.665	0.865	0.953	0.383

Table 1. The NP magic numbers for $\Delta C_{7\gamma}(\mu_b)$ defined in Eq. (5.6). For completeness, we include in the last column the case of a flavour-violating Z . The analogous table for $\Delta C_{8G}(\mu_b)$ is provided in Appendix A.

- 1) Similarly to the SM, the magic numbers κ_7 and κ_8 suppress the initial values $\Delta C_{7\gamma}(\mu_H)$ and $\Delta C_{8G}(\mu_H)$. This is due to the QCD RG evolution running down to μ_b . Furthermore, the suppression of $\Delta C_{7\gamma}(\mu_H)$ increases with μ_H .
- 2) As in the SM, provided $\Delta^{AB}C_2^f(\mu_H)$ and $\Delta^{AB}\hat{C}_2^d(\mu_H)$ are sufficiently larger than $\Delta C_{7\gamma}(\mu_H)$, the additive QCD corrections stemming from the mixing of the neutral current-current operators into the dipole operators are dominant. Furthermore the QCD factors κ_i increase in most cases with μ_H . The most prominent is the coefficient $\hat{\kappa}_{LR}^d$ which could even be of $\mathcal{O}(1)$, but also $\kappa_{LR}^{u,c}$, $\hat{\kappa}_{LL}^d$ and $\kappa_{LL,LR}^{s,b}$ are sizable. However, values of the corresponding initial conditions could compensate these coefficients, as in the case of small couplings among SM fermions.
- 3) The SM and NP primed dipole Wilson coefficients, $C_i'(\mu_W)$ and $\Delta C_i'(\mu_H)$, are suppressed by m_s/m_b and turn out to be numerically negligible. On the other hand, this suppression is absent in the contributions of the primed neutral current-current operators $Q^{nn'}$ and therefore they should be considered in the determination of the branching ratio.

5.2 Three classes of models

The results for κ_i shown in Tab. 1 are model-independent and hold for all models in which the neutral gauge bosons have flavour violating couplings to fermions as discussed in Sect. 2. On the other hand, the initial conditions of the Wilson coefficients are model-dependent. However, in spite of the large variety of models it is possible to distinguish three main classes, even if hybrid situations are also possible:

- 1) Models without exotic fermions. This is the case of models that do not aim at explaining the SM fermion masses and mixings or models that do provide such an explanation but without exotic fermions. For instance, Z' constructions in which the theory is anomaly free without extending the fermion spectrum of the SM fall into this class. This is possible when the Green–Schwarz mechanism is implemented in the theory or when the generator of the additional $U(1)$ is a linear combination of the hypercharge Y and $B - L$, where B is the baryon number and L is the lepton number.

For all models of this class, the new diagrams with virtual exotic fermions do not contribute. On the other hand, the contributions from the exchange of the light down-type quarks, the neutral current-current operators and the corresponding QCD evolution are still present. As illustrated in our numerical analysis of Sect. 5.4, the light-quark contributions turn out to be negligible in the whole parameter space.

- 2) Models in which the SM fermion-mass patterns are governed by exotic fermions through a see-saw mechanism as illustrated in Sect. 2. In this case, the couplings of the neutral gauge bosons to SM and exotic fermions, described by Eqs. (2.12)–(2.15), are suppressed by $\sin \theta_{L_k, R_k}$ given in Eqs. (2.7) and (2.8). In the specific limit of $m'_{F_k} \gg m_{f_k}$ and $M_1^D \approx M_2^D \equiv M^D$ the suppression in $\sin \theta_{L_k, R_k}$ is approximately of order $\mathcal{O}\left(\sqrt{m_{f_k}/m'_{F_k}}\right)$.

In order to illustrate the size of the contributions from the new diagrams of Sect. 3, we redefine the couplings $C_{L,R}^{ski}$ and $C_{L,R}^{bki}$ to exhibit the dependence on the suppressing factor. Without loss of generality, we can approximately write

$$C_{L,R}^{ski} \simeq \sqrt{\frac{m_s}{m'_{F_k}}} \tilde{C}_{L,R}^{ski}, \quad C_{L,R}^{bki} \simeq \sqrt{\frac{m_b}{m'_{F_k}}} \tilde{C}_{L,R}^{bki}. \quad (5.7)$$

As a result, for the case of arbitrary A_{H_i} and F_k , the expressions on the right-hand side of Eqs. (3.9) and (3.10) are given by

$$\begin{aligned} \Delta^{LL} C_{7\gamma}^{\text{heavy}}(\mu_H) &\simeq -\frac{1}{6} \sum_{i,k} \frac{g_{H_i}^2}{g_2^2} \frac{M_W^2}{M_{A_{H_i}}^2} \frac{\sqrt{m_s m_b}}{m'_{F_k}} \frac{\tilde{C}_L^{ski*} \tilde{C}_L^{bki}}{V_{ts}^* V_{tb}} \left(C_{8G}^{SM}(x_{ki}) + \frac{1}{3} \right), \\ \Delta^{LR} C_{7\gamma}^{\text{heavy}}(\mu_H) &\simeq -\frac{1}{6} \sum_{i,k} \frac{g_{H_i}^2}{g_2^2} \frac{M_W^2}{M_{A_{H_i}}^2} \sqrt{\frac{m_s}{m_b}} \frac{\tilde{C}_L^{ski*} \tilde{C}_R^{bki}}{V_{ts}^* V_{tb}} C_{8G}^{LR}(x_{ki}), \end{aligned} \quad (5.8)$$

and $\Delta^{(LL,LR)} C_{8G}^{\text{heavy}}(\mu_H) = -3\Delta^{(LL,LR)} C_{7\gamma}^{\text{heavy}}(\mu_H)$. Similar expressions can be written for the primed contributions. Notice that the m'_{F_k}/m_b enhancement is completely removed from the LR (RL) Wilson coefficient and is replaced by the suppressing factor $\sqrt{m_s/m_b}$. The only dependence on m'_{F_k} is in the loop factor $C_{8G}^{LR}(x_{ik})$. On the other hand, an extra inverse power of m'_{F_k} is now present in the LL (RR) contribution, which further suppresses this term.

As we shall explicitly demonstrate in Sect. 5.4 these contributions turn out to be negligible with respect to those from the neutral current-current operators. The

same holds for the light-quark contributions. Hence, when dealing with this class of models, we simply neglect the contributions of exotic fermions and therefore, as in 1), only neutral current-current operators and their mixing with dipole operators are relevant.

- 3) Models with exotic fermions, in which the definition of their couplings to gauge bosons and SM fermions is independent of the mechanism of the SM fermion mass generation. For example, this is the case of models in which these couplings do not arise from the standard kinetic terms of Eq. (2.9). As a result, $\sin \theta_{L_k, R_k}$ can in general be much larger than in the previous classes of models.

In this case, the expressions in Eqs. (3.9) and (3.10) do not suffer from the additional suppressions of Eq. (5.8) and in particular the LR (RL) contribution is strongly enhanced by the factor m'_{F_k}/m_b . For the models of this class, the contribution of exotic fermions is the dominant one and all the other contributions can be safely neglected.

We summarise the relevant features of the models in these three classes in Tab. 2.

Classes of models	Q_D^{heavy}	Q_D^{light}	$Q^{nn}, Q^{nn'}$
1) without exotic fermions	Absent	Negligible	Dominant
2) with exotic fermions and see-saw	Negligible	Negligible	Dominant
3) with exotic fermions but without see-saw	Dominant	Negligible	Negligible

Table 2. Summary of the different classes of models and the corresponding NP contributions to $b \rightarrow s\gamma$: contributions from the exchange of heavy exotic quarks Q_D^{heavy} , from the exchange of SM down-type quarks Q_D^{light} , and from neutral current-current operators Q^{nn} and $Q^{nn'}$.

5.3 Model-independent constraints

It has been pointed out in [62–67] that considering the measurements of $\overline{B} \rightarrow X_s \ell^+ \ell^-$ [68, 69] the sign of $C_{7\gamma}(\mu_b)$ in the presence of NP is likely to be the same as in the SM. This provides a first model-independent constraint for NP contributions: since $C_{7\gamma}^{SM}(\mu_b)$ is negative, $\Delta C_{7\gamma}(\mu_b)$ should also be negative in order to soften the tension between the central values of the SM prediction and the experimental determination of $Br(\overline{B} \rightarrow X_s \gamma)$. This holds not only for contributions from unprimed operators, but also when considering the primed ones due to the relative suppression of $C'_{7\gamma}(\mu_b)$ with respect to $C_{7\gamma}(\mu_b)$, as explicitly confirmed by our numerical analysis.

The sign of the NP contributions is determined by the product of the initial conditions and the QCD magic numbers in Tab. 1. The signs of the κ_i factors are fixed solely by the QCD running, while the sign of the initial conditions $\Delta C_i(\mu_H)$ depends on the couplings of the new neutral gauge bosons to the fermions.

In particular, for a model in the first two classes, this constraint translates into the requirement that the second line on the right-hand side of Eq. (5.6) should be negative,

namely that:

$$\Delta C_{7\gamma}(\mu_b) \simeq \sum_{\substack{A=L,R \\ f=u,c,t,d,s,b}} \kappa_{LA}^f \Delta^{LA} C_2^f(\mu_H) + \sum_{A=L,R} \hat{\kappa}_{LA}^d \Delta^{LA} \hat{C}_2^d(\mu_H) < 0, \quad (5.9)$$

where only the couplings listed in Eq. (2.12) are involved.

On the other hand, when a model belongs to the third class, the first line on the right-hand side of Eq. (5.6) must be negative,

$$\Delta C_{7\gamma}(\mu_b) \simeq \kappa_7 \Delta C_{7\gamma}(\mu_H) + \kappa_8 \Delta C_{8G}(\mu_H) < 0. \quad (5.10)$$

This puts a constraint on the couplings of Eqs. (2.14) and (2.15).

Once the coupling constants are chosen such that the NP contributions have the same sign as the SM one, it is possible to further constrain the parameter space by requiring that the predicted branching ratio should not exceed the experimental bound. From Eq. (5.3) we find the constraint

$$-\Delta C_{7\gamma}(\mu_b) + 1.4 \left(|\Delta C_{7\gamma}(\mu_b)|^2 + |\Delta C'_{7\gamma}(\mu_b)|^2 \right) \lesssim 4.2(6.1) \times 10^{-2}, \quad (5.11)$$

corresponding to the 1σ (2σ) departure from the experimental value, Eq. (5.2).

It is straightforward to apply these constraints to all models belonging to one of the three classes discussed above. It is not possible to obtain more insight without specifying a particular model (an analysis will follow in [45]). However, in the next section, we provide a simplified representative for each class of models and show how the constraints discussed so far apply to each of them.

5.4 Toy-model examples

For each class discussed above, we consider a toy-model in order to justify the approximations made in the previous sections and to exemplify the application of the model-independent constraints.

Classes 1) and 2): Here, the relevant initial conditions are those presented in Eqs. (4.16) and (4.17). The coupling constants entering these expressions are $C_{L,R}^{sb}$, $C_{L,R}^{ff}$, $C_{L,R}^{sd}$ and $C_{L,R}^{bd}$, with $f = u, c, t, d, s, b$. To simplify the analysis, we assume that all flavour-violating couplings with a strange flavour are equal to C_{FV}^s , the two bottom couplings $C_{L,R}^{bd}$ equal to C_{FV}^b and all the flavour conserving ones equal to C_{FC} . Furthermore, we take $V_{ts} = -0.04047$, $V_{tb} = 0.999146$ [70], assume that $g_H = g_2$, and consider only one heavy neutral gauge boson. In this way we have defined a toy model with only four free parameters: three coupling constants and the mass of the neutral gauge boson.

In Fig. 2 on the left we show the breakdown of $C_{7\gamma}(\mu_b)$ in its different contributions as a function of the coupling constants $C_{FV} \equiv C_{FV}^s = C_{FV}^b = C_{FC}$, for $M_{A_H} = 1$ TeV. For completeness, we also show the exotic-quark contributions for $m'_F = 10$ TeV. As expected from the discussion in Sec. 5.2, all the NP contributions apart from the

neutral current-current are negligible, justifying our approximations in the previous section. We remark that $C'_{7\gamma}(\mu_b)$ almost coincides with the neutral current-current contribution (red line). In Fig. 2 on the right we show the value of the coupling constants C_{FV} , as a function of M_{A_H} , for which the bound in Eq. (5.11) is saturated at the 1σ and 2σ level. As we can see, for small values of M_{A_H} the couplings are constrained to small values.

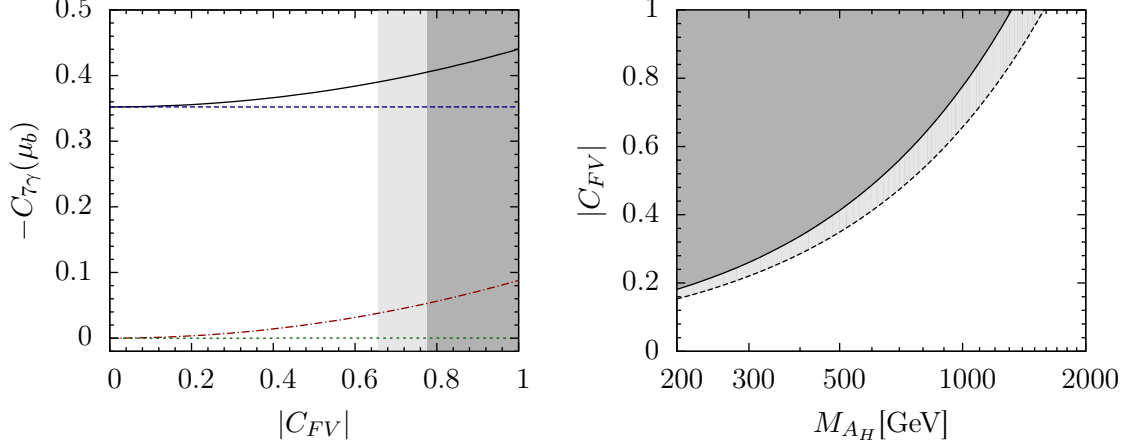


Figure 2. On the left, the different contributions to $C_{7\gamma}(\mu_b)$ (solid line) are plotted as functions of C_{FV} : in blue the SM contribution, in red the neutral current-current contribution, in green the overlapping contributions from exotic and SM down-type quarks. The shadowed region is excluded imposing the bound in Eq. (5.11) at 1σ (lighter) and 2σ (darker). On the right, we show the value of C_{FV} as a function of M_{A_H} for which the bound in Eq. (5.11) is saturated. Again the shadowed region represents the excluded values.

In Fig. 3 we separately present the implementation of the two model-independent constraints, for $M_{A_H} = 1$ TeV. On the left, the constraint on the sign of $\Delta C_{7\gamma}(\mu_b)$ mostly reduces the parameter space to cases in which the coupling constants C_{FV}^s and C_{FV}^b have opposite signs. In Fig. 3 on the right, the bound from the branching ratio also applies, further constraining the parameter space.

Class 3): We consider here the case with only one heavy neutral gauge boson and one exotic fermion. The relevant initial conditions are those in Eqs. (3.9) and (3.10). The coupling constants which enter these expressions are $C_{L,R}^{sF}$ and $C_{L,R}^{bF}$. Fixing $g_H = g_2$, $M_{A_H} = 1$ TeV and $m'_F = 10$ TeV, and identifying the coupling constants $C_{FV} \equiv C_{L,R}^{sF} = C_{L,R}^{bF}$, we illustrate in Fig. 4 on the left that the dominant NP contributions stems solely from exotic quarks. For a complete comparison, we also plot the contributions from light quarks and neutral current-current operators, adopting the same conventions as for Fig. 2.

In Fig. 4 on the right we show the value for the coupling constants C_{FV} as function of M_{A_H} for which the 1σ and 2σ bounds in Eq. (5.11) are saturated. As we can see the constraint on the couplings is very strong and only a small part of the shown parameter space survives even for large values of M_{A_H} .

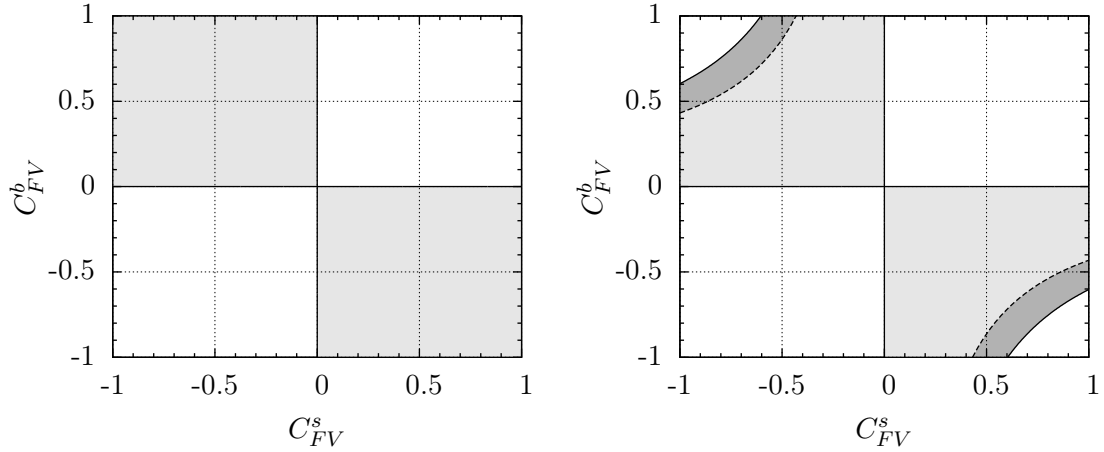


Figure 3. On the left (right), the points satisfying $\Delta C_{7\gamma}(\mu_b) < 0$ (Eq. (5.11)) in the plane C_{FV}^s vs. $C_{FV}^b (\equiv C_{FC})$. The shadowed regions now represent the values for which the bounds are passed: lighter (darker) intensity refers to $-4.2(6.1) \times 10^{-4}$.

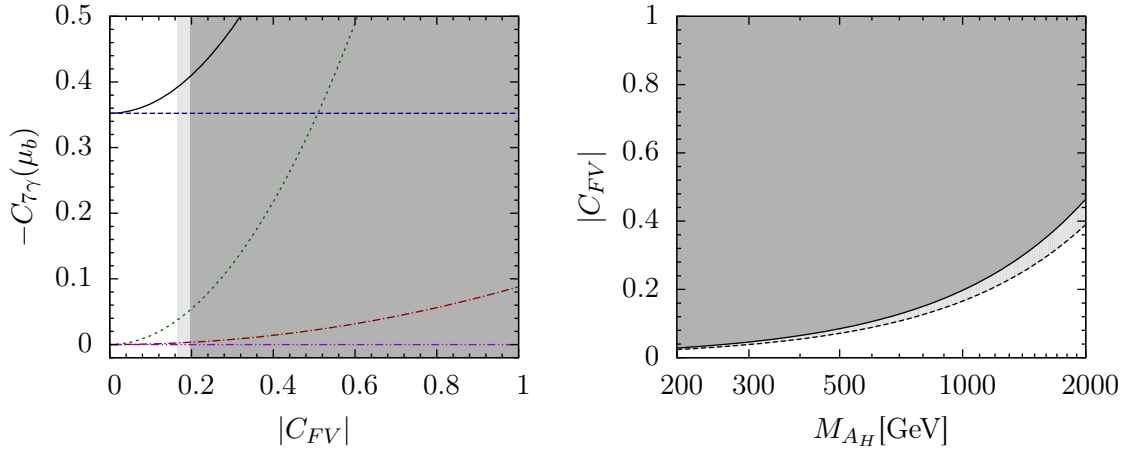


Figure 4. A similar description to Fig. 2 applies. Here, however, the green (purple) line refers to the exotic-quark (SM-quark) contributions.

In Fig. 5 on the left (right) we show how the couplings are constrained by the expression in Eq. (5.10) (Eq. (5.11)): a negative value for $\Delta C_{7\gamma}(\mu_b)$ is only recovered when both C_L^{sF} and $C_{L,R}^{bF}$ have the same sign, while the lower bound in Eq. (5.11) provides a very strong constraint on the parameters of the model.

6 Conclusions

Extensions of the SM in which the gauge group is enlarged by additional symmetries are attractive, because they provide an explanation of the flavour puzzle and predictions testable at colliders. In these models heavy exotic fermions are usually introduced in order to cancel possible anomalies and to justify the observed SM fermion spectrum through the

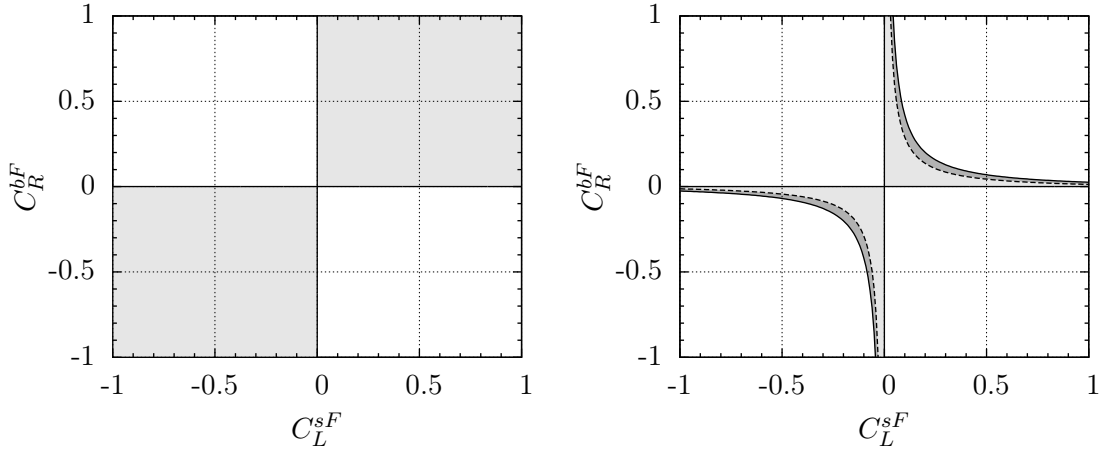


Figure 5. On the left (right), the points satisfying $\Delta C_{7\gamma}(\mu_b) < 0$ (Eq. (5.11)) in the plane C_L^{sF} vs. C_R^{bF} , for $C_L^{sF} \in [-1, 1]$. The shadowed regions are defined in Fig. 3.

see-saw mechanism. On the other hand, the presence of new heavy neutral gauge bosons and exotic fermions in principle translates into NP contributions to FCNC processes.

In this paper, we have pointed out two new contributions to the $\bar{B} \rightarrow X_s \gamma$ decay that arise in such models. The first is generated through one-loop diagrams mediated by neutral gauge bosons and exotic down-type quarks. The relevance of this contribution depends on the strength of the left-handed and right-handed flavour violating couplings of the neutral gauge bosons to the SM and the exotic quarks. Analogous effects can be present in other processes like $\mu \rightarrow e \gamma$ and $t \rightarrow c \gamma$ as well as flavour conserving observables like $(g - 2)_\mu$ and EDMs, in which dipole operators play the dominant role.

The second contribution is due to the presence of neutral current-current operators, mediated by neutral gauge bosons, and arises only if the neutral gauge bosons have flavour violating couplings to the SM quarks. Through the QCD mixing with the magnetic dipole operator $Q_{7\gamma}$, these new neutral current-current operators contribute to $\bar{B} \rightarrow X_s \gamma$. To our knowledge these QCD effects have been calculated here for the first time. Furthermore, our QCD analysis of the mixing among neutral current-current, QCD penguin and gluonic dipole operators could also be relevant for other processes, such as non-leptonic two-body B decays, and other observables, like ϵ'/ϵ .

Beside these new contributions, we have also considered the contributions arising from one-loop diagrams with the exchange of neutral gauge bosons and SM down-type quarks, that have been only partially analysed in the literature.

We have studied the impact of all these contributions in a model-independent approach and summarised the resulting constraints in Eqs. (5.9)–(5.11). We have presented these expressions in such a manner that in order to test a specific model, it is sufficient to specify only the values of the couplings of the neutral gauge bosons to SM and exotic quarks and their masses. In particular, it is not necessary to repeat the QCD analysis.

A detailed application of this study on a concrete NP scenario is in progress. Here, without entering into details of a particular model, we have described three representative

classes of models and discussed the relevance of these contributions. For models in the first class, the SM spectrum is enriched only by the neutral gauge bosons, but no exotic quarks are present. In this case the contributions from the exotic quarks are absent, but those from the neutral current-current operators turn out to have a potentially observable effect on the branching ratio of $\bar{B} \rightarrow X_s \gamma$. The value of the masses of the neutral gauge bosons and the strength of their flavour violating couplings to SM fermions determine the relevance of this effect.

The second class accounts for models in which the SM fermion masses m_f are explained through the see-saw mechanism with heavy exotic fermions of masses m'_F . In this case, the couplings of neutral gauge bosons to SM and exotic fermions are suppressed by terms m_f/m'_F . Therefore, the contributions to $Br(\bar{B} \rightarrow X_s \gamma)$ from the exchange of exotic quarks turn out to be negligible with respect to those from the QCD mixing of the neutral current-current and magnetic dipole operators.

The models in the third class are characterised by the presence of heavy exotic fermions, which either provide the SM fermion masses through a different mechanism than the see-saw or do not participate at all in the explanation of the SM flavour spectrum. In this case, no suppression occurs in the couplings of neutral gauge bosons to SM and exotic fermions and the contributions to $Br(\bar{B} \rightarrow X_s \gamma)$ from exotic quarks are enhanced by terms m'_F/m_b , where m_b is the mass of the bottom quark. These contributions dominate over all the others.

For all the models in the three classes, the contributions from SM down-type quarks turn out to be subdominant in the whole parameter space.

Our analysis shows once more how FCNC processes can put constraints on beyond-SM constructions even before the discovery of new particles in high-energy processes.

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A The $\bar{B} \rightarrow X_s g$ decay

Similarly to Eq. (5.4) we also evolve ΔC_{8G} down to $\mu_b = 2.5$ GeV to obtain

$$\begin{aligned} \Delta C_{8\gamma}(\mu_b) = & \rho_7 \Delta C_{7\gamma}(\mu_H) + \rho_8 \Delta C_{8G}(\mu_H) + \\ & + \sum_{\substack{A=L,R \\ f=u,c,t,d,s,b}} \rho_{LA}^f \Delta^{LA} C_2^f(\mu_H) + \sum_{A=L,R} \hat{\rho}_{LA}^d \Delta^{LA} \hat{C}_2^d(\mu_H), \end{aligned} \quad (\text{A.1})$$

with the NP magic numbers ρ_i listed in Tab. 3.

μ_H	200 GeV	1 TeV	5 TeV	10 TeV	M_Z
ρ_7	0	0	0	0	0
ρ_8	0.568	0.504	0.456	0.439	0.607
$\rho_{LL}^{u,c}$	-0.124	-0.138	-0.147	-0.150	-0.115
ρ_{LL}^t	-0.015	-0.033	-0.046	-0.050	–
ρ_{LL}^d	-0.124	-0.138	-0.147	-0.150	-0.115
$\rho_{LL}^{s,b}$	-0.243	-0.279	-0.307	-0.318	-0.222
$\hat{\rho}_{LL}^d$	-0.119	-0.141	-0.160	-0.168	-0.107
$\rho_{LR}^{u,c}$	0.184	0.229	0.270	0.287	0.160
ρ_{LR}^t	0.015	0.037	0.055	0.062	–
ρ_{LR}^d	0.184	0.229	0.270	0.287	0.160
$\rho_{LR}^{s,b}$	0.311	0.382	0.447	0.474	0.272
$\hat{\rho}_{LR}^d$	-0.064	-0.052	-0.034	-0.025	-0.067

Table 3. The NP magic numbers for $\Delta C_{8G}(\mu_b)$ in Eq. (A.1). For completeness, we include in the last column the case of a flavour-violating Z .

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